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Market participants use several definitions of *private debt*, most often limited to describing general loans to small- and medium-sized corporations. In this article, private debt is any form of privately arranged financing for non-household borrowers. While our definition contains financing for general corporate use, the key areas of interest to professional investors are specialty object financing (such as for transport or commercial real estate) and project financing. Although private debt has been traditionally almost exclusively the domain of bank financing, there have also been non-bank private debt transactions, including sub-sovereigns generating financing directly from institutional investors, or insurance companies originating retail mortgages. However, most debt transactions in Europe have been private bank loans. So far, only large corporations and frequent borrowers (such as sovereigns or large sub-sovereigns) have used the public market to raise debt financing. This is due to the significant fixed costs of arranging public offerings and ongoing auditing, as well as reporting and compliance requirements.

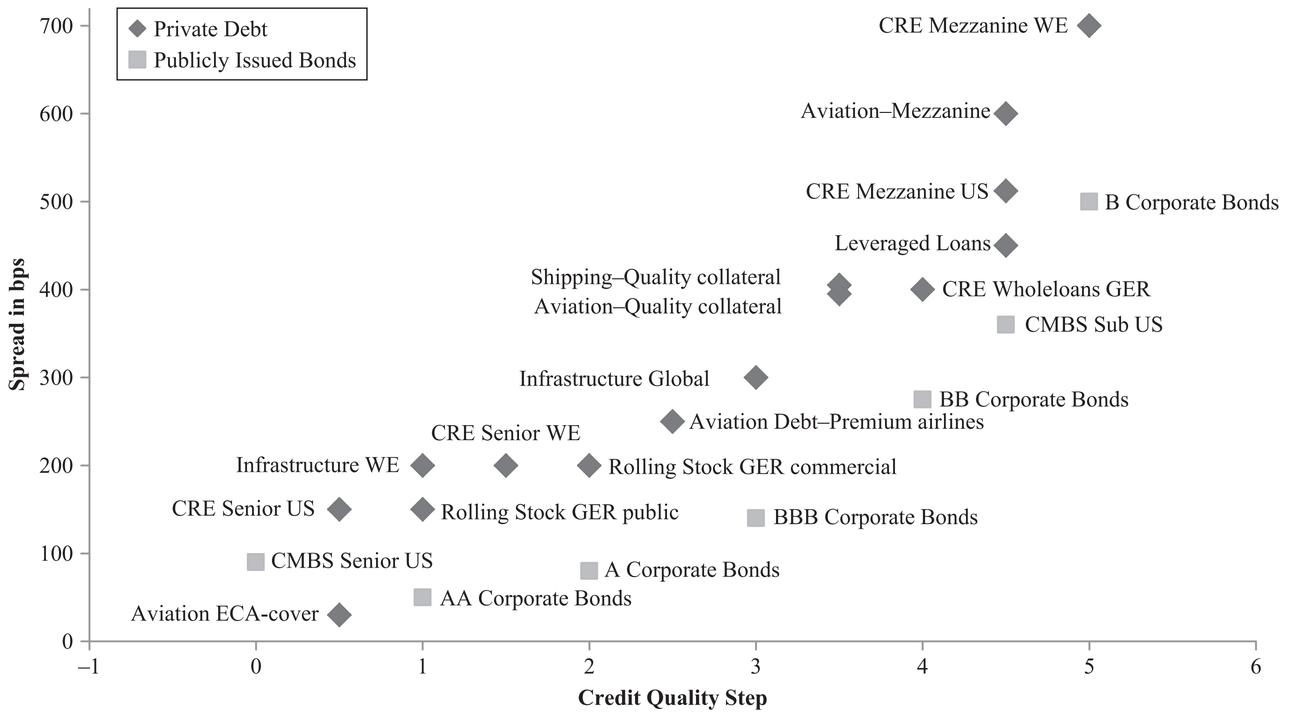
Due to the private nature of the transactions, obtaining data on yields, credit quality, defaults, and returns, for example, is difficult. Exhibit 1 provides an estimate of spreads and credit quality for selected private debt market segments based on recently observed transactions and deal proposals. The graph also shows the spread for publicly issued and actively traded bonds for comparison.

There is a noticeable and consistent difference between the spreads of private debt and bonds of comparable credit quality. The average spread difference is 140 bps, with a standard error of 15 bps, suggesting that private debt may offer higher expected returns than bonds of comparable credit quality. If private debt and publicly issued debt have similar expected credit losses, private debt would offer statistically and economically superior expected returns. The rating agencies' default and credit loss analyses suggest that expected loss is, in fact, lower for private debt. Thus, the spread difference above is probably a conservative estimate of the difference in expected returns.

In the academic literature, the level of illiquidity in private markets is related to several factors, including asymmetric information between investors and companies, low frequency of over-the-counter transactions, search frictions to find trading partners, and exogenous transaction costs of processing the trade. For example, Vayanos and Wang [2012] explain that private information is a potential cause of illiquidity because expected returns increase when information is not disseminated evenly among market participants. De Jong and Driessen [2013] claim that, when investors differ in their expected trading

Exhibit 1

Spread of Private Debt and Comparable Liquid Bonds



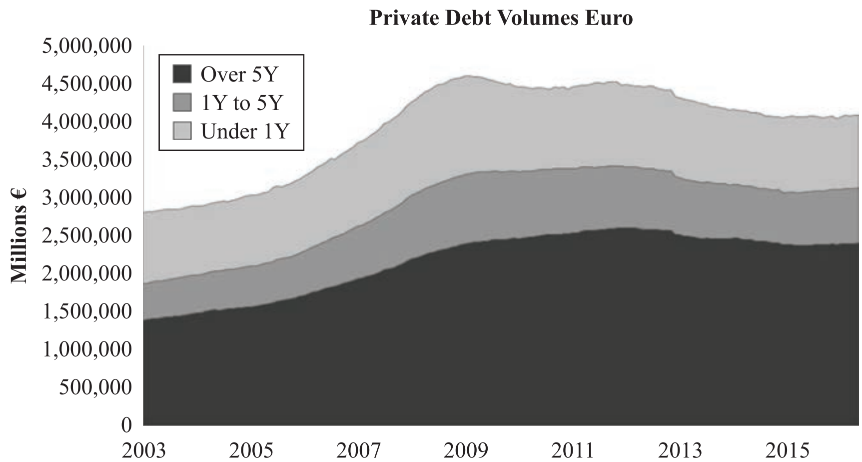
Notes: Spread figures for private debt are industry expert estimates (WE stands for Western Europe, GER for Germany, ECA for Export Credit Agency, CRE for Commercial Real Estate). Spread figures for bonds are from iBoxx EUR Corporate Indices for high grade instruments and from Merrill Lynch Indices for sub-investment grade instruments.

Source: Prime Capital, as of July 31, 2016.

horizon (e.g., investors preferences are heterogeneous), the required liquidity premium increases with the expected holding period. Similarly, according to Ang, Papanikolaou, and Westerfield [2014], investors demand a liquidity premium as compensation for not being able to trade for the duration of their expected trading horizon. In general, it is possible to distinguish between two types of liquidity premia (Lou and Sadka [2011]; Khandani and Lo [2011]): a liquidity level premium, which compensates for the average illiquidity of a security (mean level of liquidity), and a liquidity risk premium, which compensates for the illiquidity risk of assets that perform poorly during systematic liquidity shocks (volatility of liquidity). Yet, it is difficult to disentangle and measure empirically these liquidity premia, mainly due to a lack of data, as highlighted also by Cornel [2017].

In this article, we consider aggregated historical data on bank loan interest rates as a proxy for private debt performance, and estimate its components by means of a multivariate regression analysis on interest rates, credit spreads, and volatility. Similarly to the findings of Kinlaw, Kritzman, and Mao [2015] for private equity investments, we detect a significant residual that remains to be explained. This residual may be related to an illiquidity or complexity premium inherent to private debt investments. Furthermore, we show the diversification opportunities that private debt offers in asset allocation thanks to its low correlation with other asset classes. We find that efficient portfolios investing in private debt are better diversified and achieve higher expected returns than portfolios that do not invest in private debt. The remainder of the article is structured as follows. In the next section, we introduce the aggregated historical data on bank loan interest rates and study their characteristics and evolution over time. In the third section, we present the performance decomposition of the loan interest rates and discuss the resulting residual that cannot be explained by interest, credit, and market

EXHIBIT 2
Monthly Volumes of Private Loans to Non-financial Corporations



Sources: ECB, MFI Interest Rate Statistics.

factors. In the fourth section, we show the diversification opportunity that private debt can present in strategic asset allocation. Finally, in the last section, we draw the main conclusions.

DATA

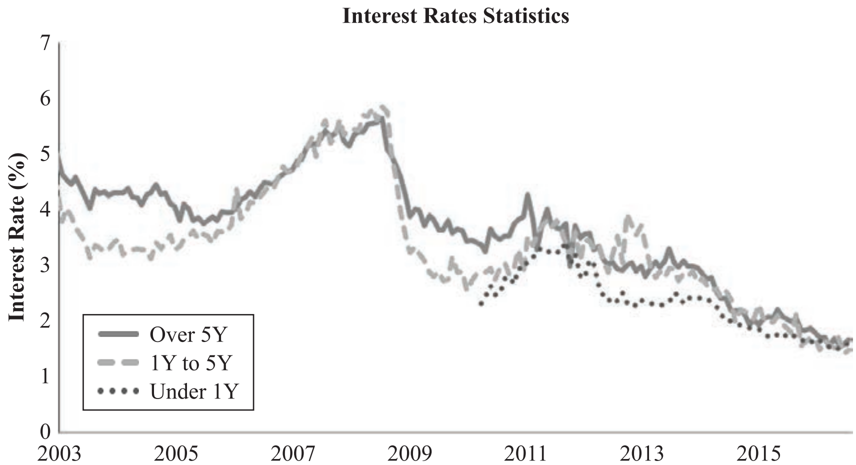
A key data source for private debt are the Bank Interest Rate Statistics maintained by the ECB and collected by the national central banks.[1] The dataset contains the volume of existing and new loans on the balance sheet of the Euro area banks and the corresponding interest rates. The data are based on a stratified sample of all loans to non-financial corporate borrowers across the entire credit quality spectrum, including, in particular, borrowers with credit qualities below investment-grade. The data are collected and published monthly. The dataset differentiates among loan types (size, maturity, and secured vs. unsecured) and broad borrower types (financial institutions, non-financial corporates, and private households). In this article, we focus on the largest loan category (over €1 million notional) to non-financial corporations, as they can be considered the most representative of institutional investors' private debt portfolios.

As at May 2016, the ECB reports that Euro area banks have a total of €4 trillion in outstanding loans to non-financial corporations. As shown in Exhibit 2, the largest share, at €2.4 trillion, has an original maturity and fixed-rate period of over five years, and short-term (<1 year maturity) loans amount to €950 billion, while intermediate-maturity (one year to five years) loans account for €730 million. Loan volumes change after the financial crisis, especially for loans with maturity of over five years, reflecting shrinking balance sheets due to the crisis. By looking at interest rates of loans with different maturities, as displayed in Exhibit 3, we cannot detect any liquidity premium as compensation for higher expected holding period, assuming that such loans have comparable credit quality, because the aggregate interest rate levels are very similar, especially in the last three years. Because the longer-dated loans are closest to private debt, the remainder of this article focuses on them.

In addition, the ECB reports the monthly new business volume of loans, that is, the notional amounts of loans extended during that month. For ease of exposition, Exhibit 4 displays the new business volume aggregated by year and the corresponding interest rate. The graph shows an increase in the volume of new loans up to the financial crisis in 2008, a marked decline over the next years, and a significant increase only in 2015. Looking at the effective interest rates corresponding to the loans, we notice the same decline after the financial crisis. A lower interest rate level on new loans may be the result of different events. First, it is a well-known hypothesis that banks resort to credit rationing rather than adapting prices to changes in credit quality. In addition, it is reasonable to assume that banks were

Exhibit 3

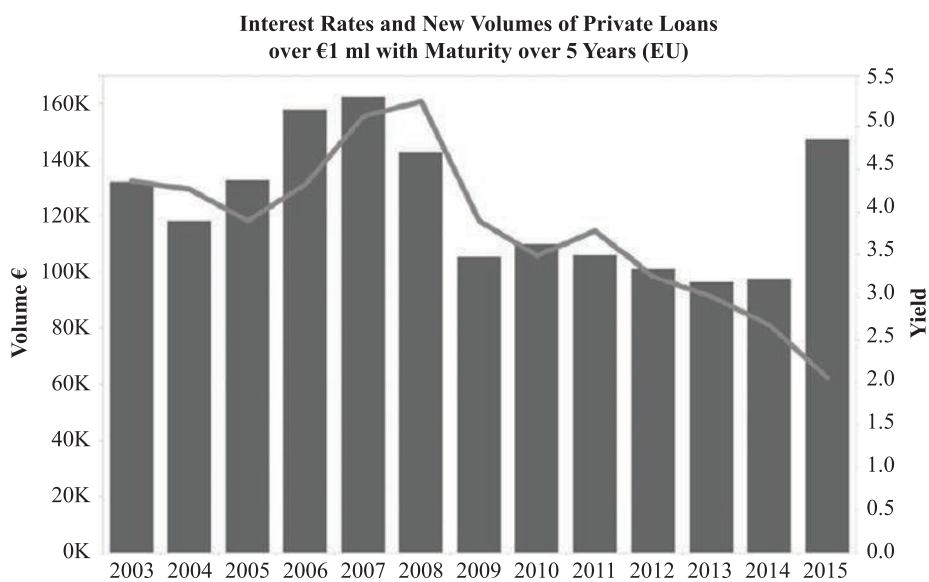
Monthly Interest Rates of Private Loans to Non-financial Corporations



Sources: ECB, MFI Interest Rate Statistics.

Exhibit 4

Monthly Interest Rates, Statistics, and Volumes of New Private Loans to Non-financial Corporations



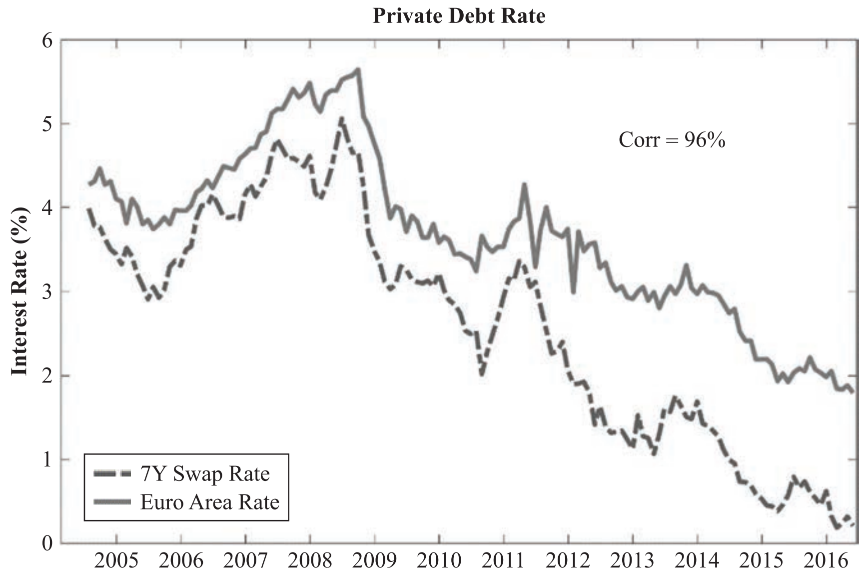
Sources: ECB, MFI Interest Rate Statistics.

severely constrained by their capital base during the crisis. Furthermore, it is quite likely that the credit quality of new borrowers (i.e., those that succeeded in obtaining financing from a bank) during the crisis is much higher than before or after. Exhibit 5 shows the monthly time-series of sample average interest rates together with the 7-years EUR swap rates; the significant correlation suggests that this maturity is a good estimate of the average new loan's maturity.

The time-series of the loan interest rates shows an expected pattern: loan rates are above swap rates and track them quite closely; loan rates increase during the

Exhibit 5

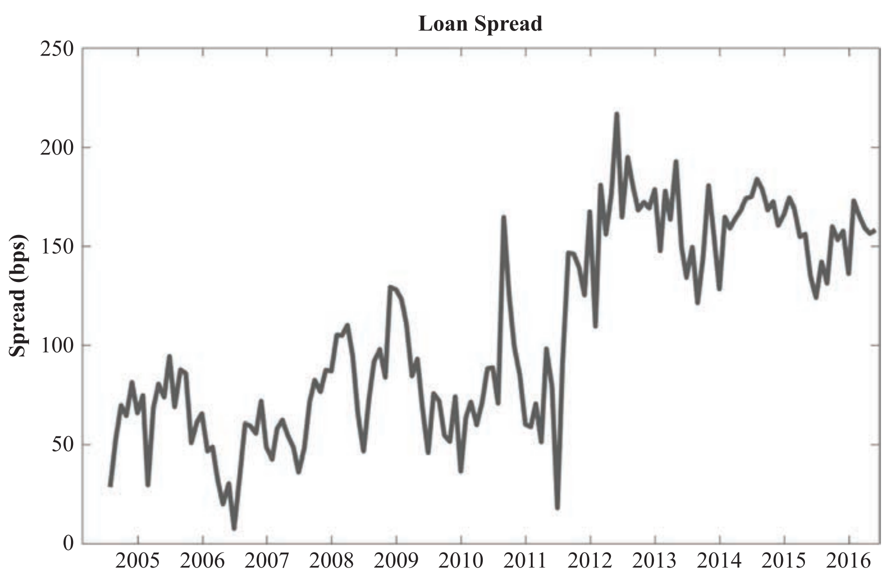
Monthly Interest Rates of Private Loans to Non-financial Corporations and 7-year Interest Rate Swap



Source: ECB and Bloomberg.

Exhibit 6

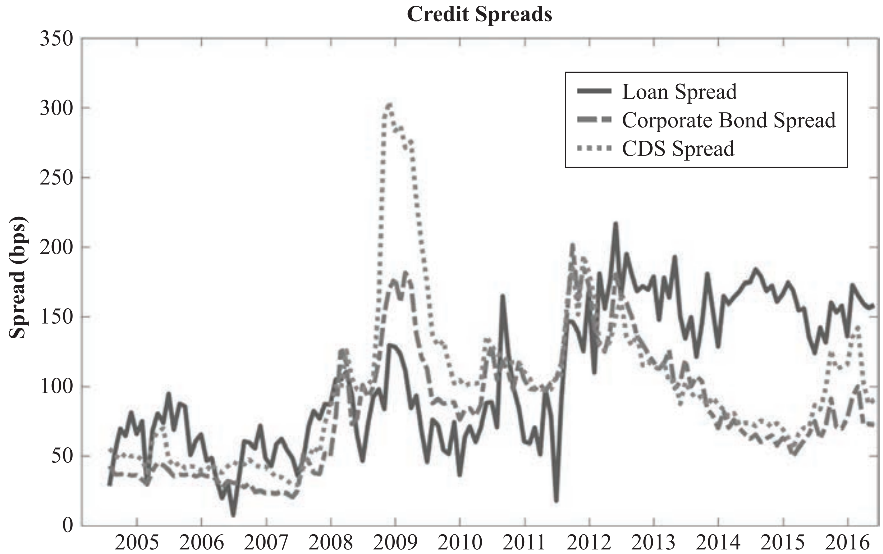
Monthly Spread between Interest Rates of Private Loans and 7-year Interest Rate Swap



financial crisis and contract in its aftermath as interest rates collapse. We remove the interest rate component by taking the difference between the loan rates and the 7-years EUR swap rates as a proxy for the loans spread. While the loan interest rates show a downward tendency after the financial crisis, the spread between loan and swap rates in Exhibit 6 shows a more complex picture: spreads exhibit an increasing trend after the financial crisis and in particular since 2011. The average monthly spread is 107 bps with a standard deviation of 50 bps.

EXHIBIT 7

Loan Spread, iBoxx Asset Swap Spread for Non-financial Investment-grade Corporate Bonds, and iTraxx Main Credit Default Swap Spread in Europe



Source: Markit.

PRIVATE DEBT PERFORMANCE DECOMPOSITION

After having removed the interest rate component, we compare the loan spread with the spreads of other credit instruments with similar risk, notably corporate bond spreads (iBoxx ASW spreads for non-financial investment-grade EUR corporate bonds), and (iTraxx Main) CDS spreads.

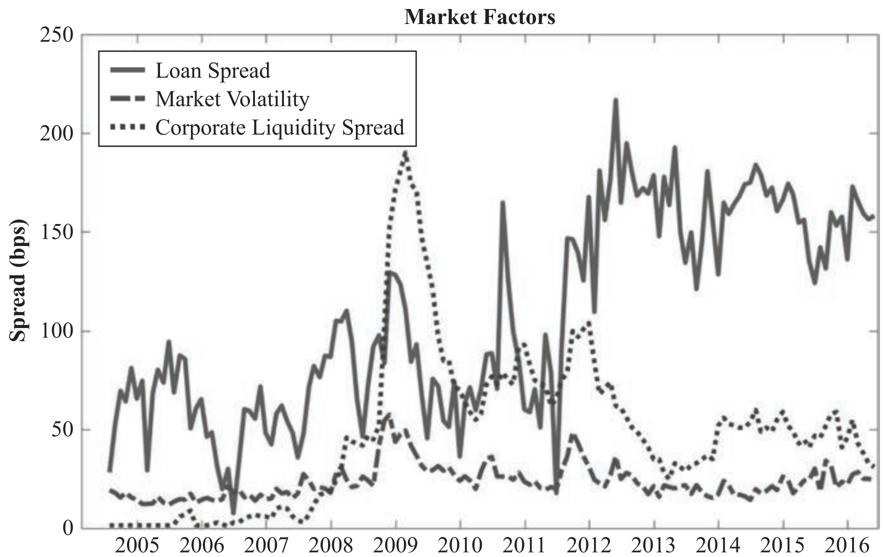
Some interesting patterns emerge by looking at the evolution of these spreads in Exhibit 7. Firstly, corporate bond spreads and CDS spreads align very well most of time, the notable exceptions being the financial crisis in 2008, when bond spreads widened, and 2015, when bond spreads also increased significantly. The excessive bond spread is usually attributed to the lower liquidity of the bonds compared to the CDS. Secondly, there is some co-movement, as one ought to expect, between loan spreads and the liquid instruments' spreads. The monthly correlation between loan spreads and bond spreads is 0.29, while the monthly correlation between loan spreads and CDS spreads is 0.46. This suggests that the loan spread and the liquid instruments' spread reflect similar pricing of underlying systematic credit risk. The correlation with bond spreads appears low, given the highly-diversified nature of both the underlying and the presumable lack of other common drivers.

At this point, however, it is important to notice a significant difference in sampling between loans and the liquid instruments. While the loan data relate to financing agreements entered into during any time of the month, the liquid instruments' spreads are sampled at the end of the month. There is thus a significant timing mismatch between the loan series and the liquid instrument series. More specifically, the loan series leads the liquid instrument series. Therefore, the estimated correlation, while already statistically and economically significant, probably understates the true correlation between loan spreads and bond spreads or CDS spreads. This implies a significant common driver, presumably credit risk pricing. However, the measured correlations are also significantly non-uniform, suggesting that there are other economic forces potentially driving loan spreads and liquid instruments' spreads. The correlation analysis points out that credit risk is one of the factors driving loan spreads. The next step is quantifying how much of the spread and its variation is due to credit risk.

Other factors that could drive loan spread performance are market volatility, quantified by the Euro Stoxx 50 Volatility Index, and corporate liquidity spread,

EXHIBIT 8

Loan Spread, Euro Stoxx 50 Volatility Index, and Liquidity Spread between the iBoxx Euro Liquid Corporate Index and the iBoxx Euro Corporate Index



computed as the difference between price levels of the iBoxx Euro Liquid Corporate Index and the iBoxx Euro Corporate Index.

These factors are mildly correlated throughout the whole sample period, as shown in Exhibit 8. Comparing them to our loan spread, we notice that they reach similar levels up to 2011. Therefore, we expect a time-varying effect of volatility and liquidity risk on loan spreads. We point out here that the corporate liquidity spread represents the difference in liquidity price of corporate bonds. Therefore, we expect to find in loan spreads a further liquidity component that compensates for the more illiquid nature of loans relative to bonds.

Private Debt Components

We observe that loan spreads are highly persistent, as suggested by a serial correlation of 0.79. Statistically, this is somewhat worrisome because it implies that regression standard errors are biased. Economically, however, it is very interesting because it suggests that the residual captures a slowly evolving (i.e., fairly stable) factor present in loan spreads. This factor may be unrelated to systematic credit risk. We account for serial correlation in the data by including in the multivariate regression an autoregressive component of the loan spread, which allows us to eliminate the persistence

EXHIBIT 9

Empirical Results from Ordinary Least Squares (OLS) Regression of Loan Spreads

		Estimate	SE	t-stat	p-value
Intercept	α	0.3932	0.391	1.006	0.316
Loan Spread$_{t-1}$	γ	0.6797	0.061	11.135	0.000
Credit$_{t-1}$	β_1	0.2645	0.093	2.846	0.005
Volatility$_{t-1}$	β_2	0.0122	0.112	0.109	0.914
Liquidity$_{t-1}$	β_3	–0.0021	0.001	–1.757	0.081

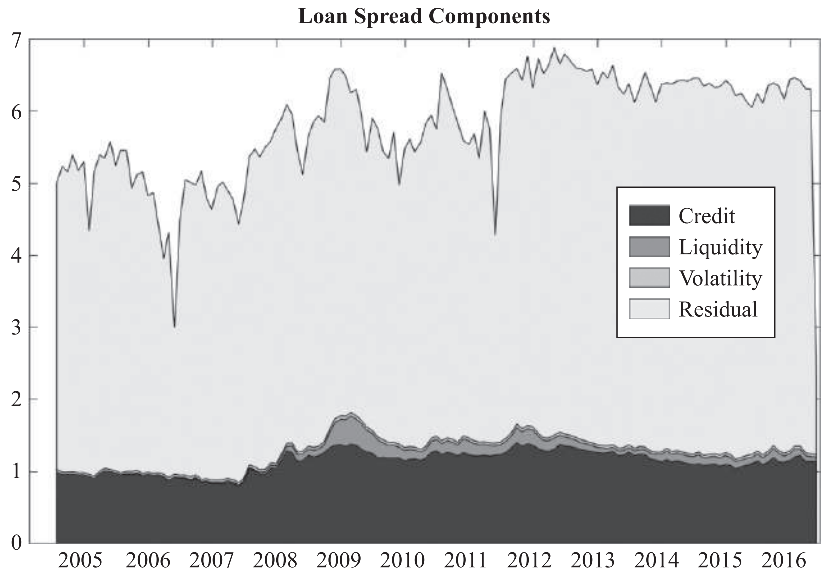
Note: At time t *on credit, volatility, and liquidity spreads at time* t – 1, *in the whole sample period: estimates, standard errors,* t*-statistics, and p-values.*

of past values, following Ang, Chen, Goetzmann, and Phalippou [2013]. We model loan spreads as follows and estimate its components via ordinary least squares (OLS):

$$\text{Loan Spread}_t = \alpha + \gamma \text{ Loan Spread}_{t-1} + \beta_1 \text{Credit}_{t-1} + \beta_2 \text{Volatility}_{t-1} + \beta_1 \text{Liquidity}_{t-1} + \varepsilon_t \quad (1)$$

From Exhibit 9, we notice that loan spreads are positively affected by the autoregressive, credit and liquidity components, as shown by the p-values in Column 5. Instead, the effect of volatility spread is not

Exhibit 10
Components of Private Debt Performance



statistically significant. The magnitude of our estimates do not change much if the variables Credit, Volatility, and Liquidity are included at time *t*. The results plotted in Exhibit 10 show that the largest share of loan spread remains unexplained, as credit risk can explain only 20%, and liquidity and volatility less than 3%. A large residual is therefore unrelated to either credit or other market factors considered here. This could be explained by the fact that loan spreads include an illiquidity premium of loans over bonds (therefore the liquidity factor does not fully capture the illiquidity of loans) and may offer further diversification opportunities. We propose to think of such residual as a *complexity premium*, inherent to the loan instrument and to the private nature of the market.

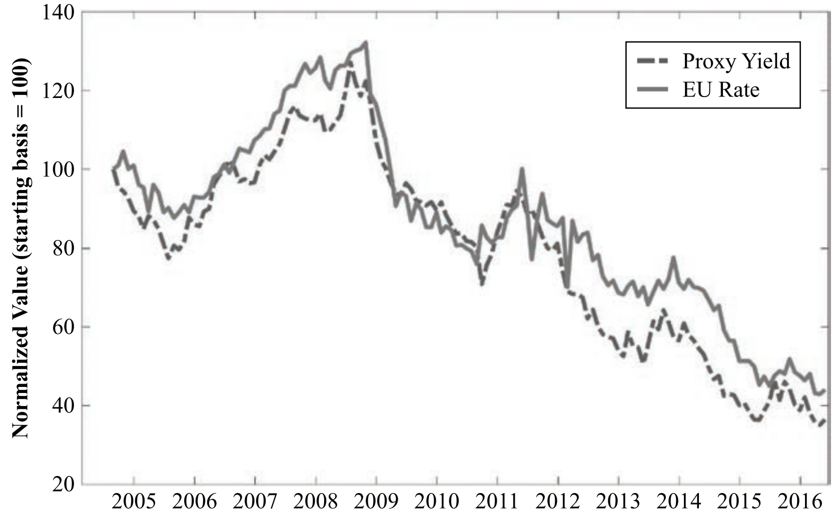
For portfolio construction, it would be useful to build a proxy for private debt returns to estimate the covariance matrix with different asset classes returns and then evaluate the potential diversification benefits. We suggest that a reasonable proxy spread for private debt could have the following three components: (i) a credit risk component reflecting the average credit quality of private debt, (ii) a liquidity component reflecting credit quality, and (iii) a volatility component. Note that the proxy spread is constructed only from components that are tradable in itself. In particular, we do not include the residual in the proxy construction. This has the advantage that the proxy is not affected by any statistical problems associated the residual, such as autocorrelation. The drawback is that the proxy possibly leaves out the most salient feature of private debt, that is, the complexity premium captured by the residual. A proxy yield may then be obtained by adding an interest rate component, for example, a swap rate of suitable maturity. Exhibit 11 shows the comparison between the original data on loan rates and our proxy, built by adding the Credit, Liquidity, and Volatility components from Exhibit 9 to the 7-year EUR swap rate. The two series exhibit a very high correlation of 97%, and show similar levels of mean (i.e., 79 for the Proxy Yield and 86 for the EU Rate) and standard deviation (i.e., 24 for the Proxy Yield and 23 for the EU Rate), as expected.

Portfolio Construction and Diversification with Private Debt

Estimates of the covariance matrix usually rely on historical total return series. Returns for credit instruments, over a certain period, are the sum of returns due to price changes, the interest paid for the period, and returns due to realized credit losses in case of a default. The price changes reflect changes in expected credit losses, that is, realized credit losses reflect only unexpected credit losses. The spread data used in this study

EXHIBIT 11

Interest Rates of Private Loans to Non-financial Corporations in Europe and Our Proxy



are roughly equivalent in nature to price data because they are monthly samples of averages of new loans. Any change in aggregate expected credit losses would be reflected in the spreads (assuming that overall characteristics of new and old loans are similar). For the dataset used in this study, only spread data are available, not realized credit losses. However, if credit losses are not too frequent or large, a return series based on spreads should yield reasonably accurate covariance estimates.

We approximate Private Debt returns $R(PD)_t$ for month t by

$$R(PD)_t = -ModDur_{t-1} \times \Delta LoanYield_t + \frac{1}{12} LoanYield_{t-1} \tag{2}$$

where $ModDur_{t-1}$ is an estimate of the modified duration of time $t-1$ and $\Delta LoanYield_t$ is the change in loan yields during the month. This approximation is based on the (well-known) Taylor series expansion of a loan's present value. The return on a loan $R(L)$ may be defined as

$$R(L)_t = \frac{V(Y(t),t) + I(t)}{V(Y(t-1),t-1)} - 1$$

where $Y(t)$ is our proxy yield at time t, $V(Y,t)$ is the value of the loan with yield Y at time t, and $I(t)$ is the interest paid on the loan between $t-1$ and t. A Taylor series expansion of $V()$ yields

$$V(Y(t),t) = V(Y(t-1),t-1)\left(1 + \frac{\partial V}{\partial Y}dY + \frac{\partial V}{\partial t}dt + \cdots\right)$$

where $\partial V/\partial Y = -ModDur$ and dY is the change in yield. The term $\partial V/\partial t$ captures the "pull-to-par" effect. Recall that our loan interest data reflect new loans, which are typically issued at par, so that $V(Y,t) = 1$. Further, the "pull-to-par" effect is 0 in such a setting. Therefore,

$$R(L)_t \approx -ModDur \times dY + D(t-1,t)\ Y(t-1)$$

where $D(t-1,t)$ is the day count fraction corresponding to the period $t-1$ to t and the timing convention of Y. In our setting of a large sample of loans and absent any further information on the periodicity of interest payments, the accrued interest method should yield a reasonable approximation.

We compare then different asset class indexes to consider possible diversification benefits in strategic asset allocation: MSCI Europe, MSCI Europe Real Estate, STOXX Europe Private Equity 20, iBOXX Corporate Bonds, iBOXX Sovereign Bonds Eurozone, and our Private Debt Proxy. The correlation matrix in Exhibit 12 suggests that Private Debt offers some diversification opportunities because it is uncorrelated to equity and real estate, negatively correlated to private equity indexes, and, not surprisingly given how we construct the proxy, positively correlated to corporate and

Exhibit 12

Correlation Matrix of Returns of MSCI Europe, MSCI Europe Real Estate, STOXX Europe Private Equity 20, iBOXX Corporate Bond, iBOXX Sovereign Bond Eurozone, and Our Private Debt Proxy

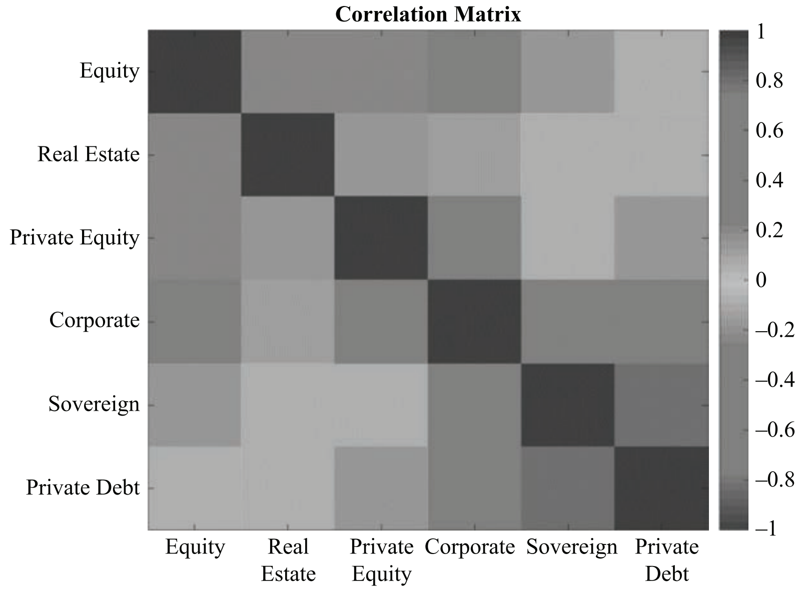


Exhibit 13

Annual Historic Return, Annual Standard Deviation, and Correlation Matrix of MSCI Europe, MSCI Europe Real Estate, STOXX Europe Private Equity 20, iBOXX Corporate Bond, iBOXX Sovereign Bond Eurozone, and our Private Debt Proxy (July 2004–May 2016)

			Correlations					
Asset Class	**Historic Return (%)**	**Standard Deviation (%)**	**Equity**	**Real Estate**	**Private Equity**	**Corporate**	**Sovereign**	**Private Debt**
Equity	3.65	19.30	1.00					
Real Estate	2.19	21.18	0.20	1.00				
Private Equity	4.61	24.91	0.20	0.13	1.00			
Corporate	4.38	3.91	0.24	0.10	0.36	1.00		
Sovereign	5.03	4.05	0.15	–0.09	–0.07	0.54	1.00	
Private Debt	5.85	4.26	0.06	–0.08	–0.18	0.49	0.80	1.00

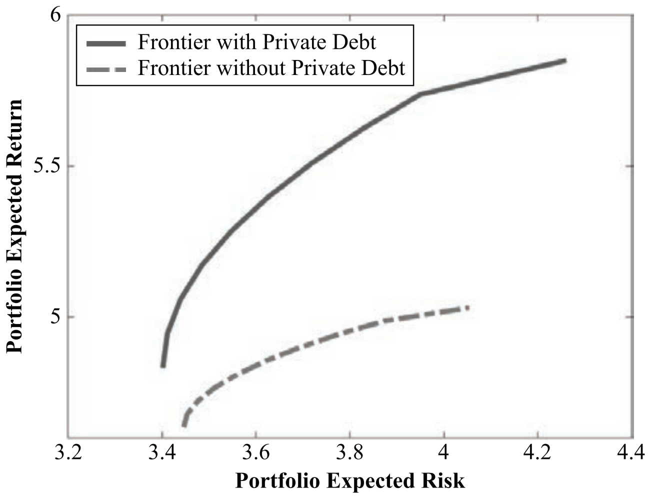
sovereign bond indexes. An examination of the annual expected return and standard deviation (Exhibit 13) also suggests that private debt is an interesting investment opportunity.

Exhibit 14 shows the mean–variance efficient frontiers obtained by including (solid) and excluding (dashed) private debt (Markowitz, [1952]). Portfolios including private debt offer higher expected returns for any given level of expected risk. This is not surprising as private debt has higher expected returns than equity and private equity, but much lower standard deviation. It is also uncorrelated with equity, real estate, and private equity, while positively correlated with corporate and sovereign bonds. We estimate ten optimal portfolios over the entire no-short-sales efficient frontiers and show how the composition changes as we move from the minimum variance portfolio to the maximum return portfolio (see Exhibit 15).

In particular, we notice that the minimum variance portfolio invests 22% of the capital in private debt;

EXHIBIT 14

No-short-sales Efficient Frontiers Including and Excluding Private Debt



increasing the risk profile we decrease the share invested in sovereign and corporate bonds in favor of private debt and private equity; the maximum return portfolio is totally invested in private debt, which is the asset class with highest return in our sample.

From a risk management perspective, it is interesting to measure how much each asset class contributes to the total risk of these portfolios (see Exhibit 16). We compare the ten portfolios on the frontier including private debt to the Equally-Weighted (EW) portfolio, the Equal Risk Contribution (ERC) portfolio, introduced by Maillard, Roncalli, and Teiletche [2010], and to a Risk Budgeting (RB) portfolio, studied by Bruder and Roncalli [2012]. The ERC is constructed such that each asset class contributes equally to the portfolio standard deviation. In the RB portfolio, the risk contribution of the asset classes is chosen a priori by the investor. We select the following risk contributions: equity 30%, real estate 10%, private equity 10%, corporate bonds 20%, sovereign bonds 20%, and private debt 10%. The risk of the resulting RB portfolio is 40% due to bonds (corporate and sovereign), 30% due to equity, and 30% due to illiquid instruments (real estate, private equity, and private debt).

Given the $d \times 1$ vector $\boldsymbol{w}$ of portfolio weights and the estimate of the covariance matrix $\hat{\Sigma}$ of d asset classes (see Exhibit 13), we compute the portfolio risk as the variance $\sigma_p^2 = \boldsymbol{w}'\hat{\Sigma}\mathbf{w}$ and the diversification indexes

$$Weight\ \ Diversification = \frac{1}{d\sum_{i=1}^{d} w_i^2}$$

$$Risk\ \ Diversification = \frac{1}{d\sum_{i=1}^{d} RC_i^2},$$

where RC_i is the risk contribution of each class i to the overall portfolio risk such that

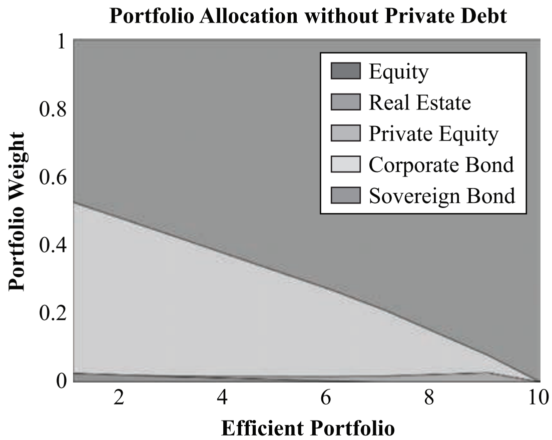
$$RC_i := w_i \frac{\delta\sigma}{\delta w_i} = w_i \frac{(\Sigma \mathrm{w})_i}{\sigma_p}$$

The *Weight Diversification* index measures diversification in terms of portfolio holdings, while the *Risk Diversification* index measures diversification with respect to risk contribution. Both indexes take values between $1/d$ and 1 for a portfolio totally concentrated in one asset class and for a fully diversified portfolio.

The efficient portfolio allocation in private debt strongly depends on the optimization inputs of the mean-variance framework, that is, expected return and risk, which are not observable ex-ante and have to be estimated from past data. Estimation errors in expected returns might have a large impact, as sample means are typically subject to large variation. Chopra and Ziemba [1993] found that errors in mean are at least 10 times as important as errors in variances. For this reason, we test the sensitivity of the diversification properties of private debt to estimation errors in mean returns, and compute the efficient portfolio allocations using lower estimates of expected returns. Exhibit 17 shows the differences between the efficient portfolios obtained by using a 1% or 2% lower annual expected return for private debt, and the original efficient portfolios reported in Exhibit 16. In the first case, the annual historic return of private debt is set to 4.85%, thus the sovereign bond class achieves the highest return in the sample. For this reason, in the left graph of Exhibit 17, we notice that private debt is substituted with sovereign bond in portfolio allocations with high risk-return profile, that is, in efficient portfolios lying on the right part of the frontier. Alternatively, in the right graph of Exhibit 17, we set the annual historic return of private debt to 3.85%, such that private equity, sovereign bonds, and corporate bonds achieve higher returns. Also in this case, we find that private debt is replaced mostly by sovereign and corporate bonds, and changes in portfolio allocation are more relevant for high risk-return profiles. Still, even if the actual annual

EXHIBIT 15

Composition of No-short-sales Efficient Portfolios Excluding (left) and Including (right) Private Debt



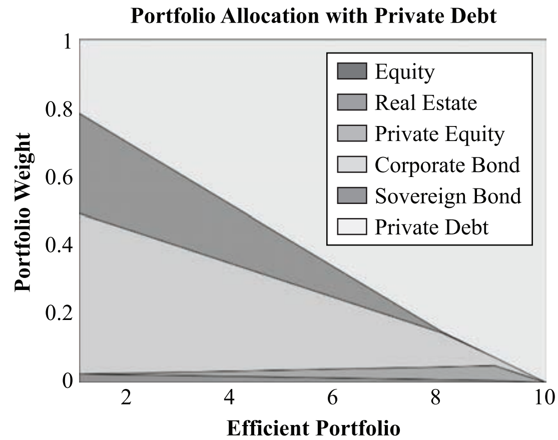


EXHIBIT 16

Portfolio Allocation and Risk Contribution of Efficient Portfolios including Private Debt, Equally-Weighted (EW), Equal Risk Contribution (ERC), and Risk Budgeting (RB) Portfolios

		Efficient Frontier											
		min $w' \Sigma w$								Max $w' \mu$			
	Asset Class	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**EW**	**ERC**	**RB**
	Portfolio Allocation (w)												
	Equity	–	–	–	–	–	–	–		–	–	0.17	0.06	0.10
	Real Estate	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	–	0.17	0.07	0.05
	Private Equity	–	0.00	0.01	0.02	0.02	0.03	0.03	0.04	0.04	–	0.17	0.06	0.04
	Corporate	0.47	0.42	0.36	0.31	0.26	0.20	0.15	0.10	0.02	–	0.17	0.24	0.29
	Sovereign	0.29	0.25	0.21	0.17	0.13	0.09	0.04	0.00	–	–	0.17	0.27	0.33
	Private Debt	0.22	0.31	0.40	0.49	0.58	0.67	0.76	0.85	0.93	1.00	0.17	0.29	0.19
	Portfolio Risk Contribution (RC)												
	Equity	–	–	–	–	–	–	–	–	–	–	0.28	0.17	0.30
	Real Estate	0.03	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.00	–	0.28	0.17	0.10
	Private Equity	–	0.00	0.01	0.01	0.02	0.02	0.02	0.03	0.03	–	0.37	0.17	0.10
	Corporate	0.47	0.40	0.34	0.28	0.22	0.16	0.11	0.07	0.01	–	0.04	0.17	0.20
	Sovereign	0.29	0.25	0.21	0.16	0.12	0.08	0.04	0.00	–	–	0.02	0.17	0.20
	Private Debt	0.22	0.32	0.43	0.53	0.64	0.73	0.82	0.90	0.96	1.00	0.01	0.17	0.10
μ_p	**Expected Return (%)**	4.83	4.94	5.06	5.17	5.28	5.40	5.51	5.62	5.74	5.85	4.29	4.80	4.69
σ_p	**Standard Deviation (%)**	3.40	3.41	3.44	3.49	3.55	3.63	3.72	3.83	3.95	4.26	7.68	4.14	4.24
	Weight Diversification	0.48	0.51	0.50	0.46	0.40	0.33	0.28	0.23	0.19	0.17	1.00	0.72	0.69
	Risk Diversification	0.48	0.51	0.49	0.43	0.36	0.29	0.24	0.20	0.18	0.17	0.56	1.00	0.83

expected returns of private debt were 1%–2% lower than our estimates, the class of private debt would play a key role in the diversification of efficient portfolios.

Independent Component Analysis

In this section, we show the diversification potential that private debt offers from a perspective of factor exposure, rather than linear correlation. We perform an independent component analysis to decompose the variance-covariance matrix $\widehat{\Sigma}$ of asset class returns into independent risk factors (Hyvärinen and Oja, [2000]). We then compare the exposure of private debt to these risk drivers to the one of the other asset classes. We find that private debt is the only class that is exposed to

EXHIBIT 17

Differences in Efficient Portfolios Obtained by Using a 1% (left panel) or 2% (right panel) Lower Annual Expected Return for Private Debt and the Original Efficient Portfolios

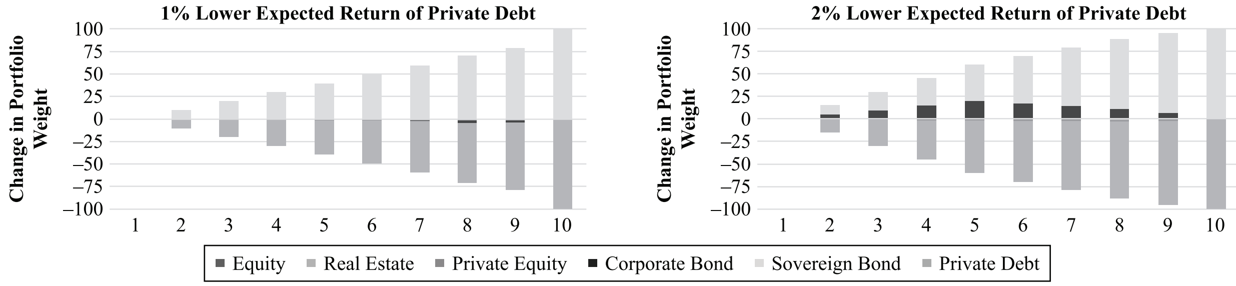
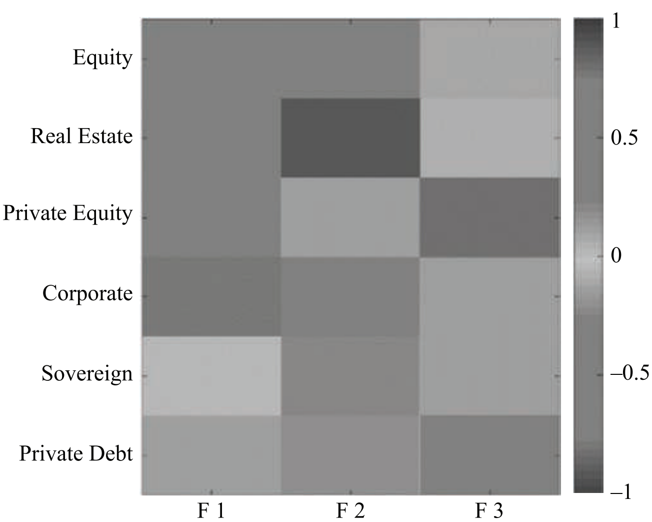


EXHIBIT 18

Correlation Matrix between Asset Class Returns and Independent Factors



one of the risk factors, reinforcing the idea that such an instrument class bears notable diversification potential.

We identify $m = 3$ risk drivers, denoted by F1, F2, and F3. F1 shows a high correlation to corporate bonds, and mild correlation to equity and private equity. F2 exhibits zero or negative correlation with all classes, but real estate. The third factor, F3, is correlated to private debt and shows highly negative correlation with private equity, while being uncorrelated to the other asset classes, as shown in Exhibit 18. Exhibit 19 shows the exposure of the ten portfolios on the efficient frontier to the three independent factors and the relative *Factor Diversification* index, computed as

$$Factor\ \ Diversification = \frac{1}{m\sum_{i=1}^{m} FC_i^2}$$

where FC_i is the contribution of each factor i to the overall portfolio risk. (see Roncalli and Weisang, [2016] for an introduction to the computation of risk contributions at the factor level.) Similar to the other diversification indexes, the *Factor Diversification* takes values between $1/m$ and 1 for a portfolio totally exposed to one risk factor and for a fully diversified portfolio. We clearly see that increasing the weight on private debt from 0.22 in portfolio 1 to 0.85 in portfolio 8, we increase the risk exposure to F3 from 0.15 to 0.37 and are thus better diversified. In fact, the factor diversification reaches the highest value of 0.99 for portfolio 8.

CONCLUSION

In this article, we consider aggregated historical data on bank loan interest rates provided by the European Central Bank as a proxy for private debt performance in Europe. We estimate its components using multivariate lagged regression analysis on loan interest rates, credit, and market factors. Our results show that there exists a complexity premium in private debt returns, which appears as a large unexplained residual. We use a proxy for private debt returns to show the diversification benefits that private debt can offer in strategic asset allocation, thanks to its low correlation with other asset classes. We find that mean–variance efficient portfolios obtained by investing in private debt are better diversified and achieve higher expected returns for any given level of expected risk than portfolios that do not include private debt. We also investigate the exposure of private debt to the independent risk factors driving

Exhibit 19

Factor Contribution of Efficient Portfolios Including Private Debt

	Portfolio Factor Contribution									
	1	2	3	4	5	6	7	8	9	10
Factor 1	0.80	0.76	0.72	0.67	0.61	0.53	0.44	0.31	0.09	–0.35
Factor 2	0.05	0.07	0.09	0.11	0.15	0.19	0.25	0.32	0.46	0.24
Factor 3	0.15	0.17	0.19	0.22	0.24	0.28	0.32	0.37	0.45	1.11
Factor Diversification	0.51	0.55	0.59	0.66	0.74	0.84	0.95	0.99	0.79	0.24

the covariance matrix of different asset classes. One risk-factor in particular appears to be specific to private debt, as it is unrelated to other asset classes, which lends further support to our findings.

Our empirical analysis might be extended in numerous ways. Firstly, we intend to explicitly address the econometric issues associated with the sampling technique and smoothing versus unsmoothing returns (Geltner [1991]; Getmansky, Lo, and Makarov [2004]). Basically, the loan data represent averages over a particular month, which implies that (i) there are certain lead-lag relations with the market data that are sampled as at month-end and (ii) that returns based on the spread series itself (rather than the proxy) appear smoothed. The results of this analysis will also allow us to investigate further the properties of the regression residual, in particular its interpretation as a complexity premium. Most importantly, it will enable us to run a well-specified portfolio analysis including the complexity premium. Secondly, in presence of more granular data on private debt returns, other approaches may be chosen in order to estimate the liquidity level and the liquidity risk premia of private debt instruments, according to Lou and Sadka [2011] and Khandani and Lo [2011]. Furthermore, different measures of diversification, other than correlation and diversification indexes, can be considered to evaluate the benefits of investing in a portfolio including private debt. Finally, a multi-asset factor framework may be used to optimal choose a portfolio allocation with private debt, according to various factor premia.

ENDNOTES

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[1]Monetary financial institutions' interest rate statistics are published in the Statistical Data Warehouse at http://sdw.ecb.europa.eu/reports.do?node=1000002883.

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